Newton's Gravitational Theory (Universal Gravitation)

But First, some History

<u>(Pre 600 BC) – Ancient Greek Model</u>

- Flat earth as the center of the universe
- Stars rotate at 15°/hr
- Sun moves 1°/day
- Stars are fastened to a gigantic crystal sphere (celestial sphere)
- Sun's path is called the <u>ecliptic</u>
- If the plane of the sun is tilted 23° from the equatorial plane, this accounts for why the sun is high in the summer and low in the winter.

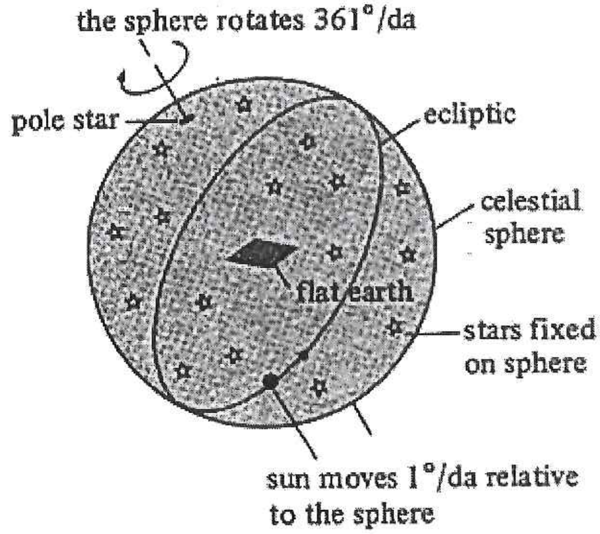


Figure 16-1 The celestial sphere with a flat earth at the center. While the stars are fixed to the celestial sphere, the sun moves along the ecliptic path (shown tinted) about 1°/da.

<u>(582-500 BC) – Pythagorus</u>

- Recognized the <u>earth was a sphere</u>. He simplified the Greek earth centered model. It was no longer necessary to adjust the axis of the celestial sphere to account for the variation of the observed height of the pole star with latitude (north-south).
- The Greeks noticed that most stars rotate at the same rate, but some rotated at rates slower than the other stars. They called these *wandering stars* or ***<u>planets</u>***.
- The planets rotate west to east but at certain times the direction is reversed. The looped part of the path is called the *<u>epicycle</u>*, and the reverse part is called the *<u>retrograde</u>* motion.

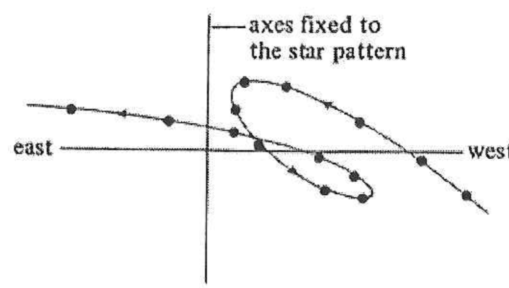
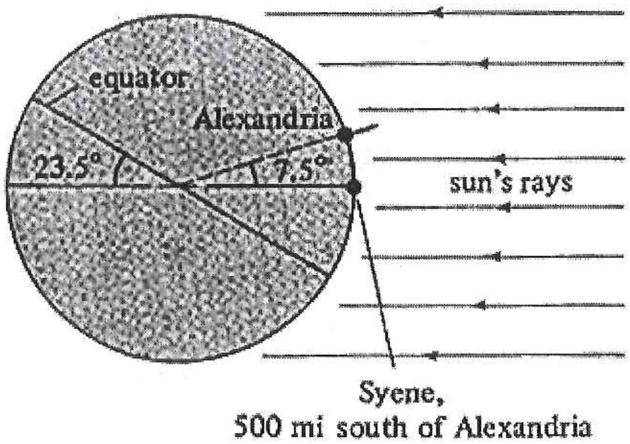


Figure 16-2 The dots show the successive positions of a planet relative to the star pattern as observed every few weeks. The tinted curve represents the path of this relative motion.

Hellenistic Age - Library in Alexandria

<u>(276-196 BC) Eratosthenes</u>

- Measured the circumference of the earth!!!
- He measured the angles the suns rays make with the vertical at noon on the first day of summer in Alexandria and Syene.

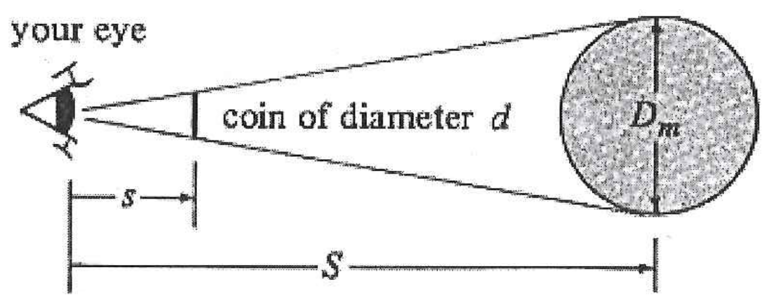


$$\frac{\underset{\text{(distance from Syene to Alexandria)}}{500 \text{ miles}}}{\text{circumference of Earth}} = \frac{7.5^\circ}{360^\circ}$$

circumference of Earth = 24,000 miles

<u>(312-230 BC) – Aristarchus of Samos</u>

- Developed a method to determine the size of the moon and sun and to find the distances to the moon and sun. His method was sound, but his measurements were incorrect.



<u>Step 1</u>

Determine the ratio of the distance to the moon divided by the diameter of the moon. $\frac{S}{D_m} = \frac{s}{d} \cong 110$

<u>Step 2</u>

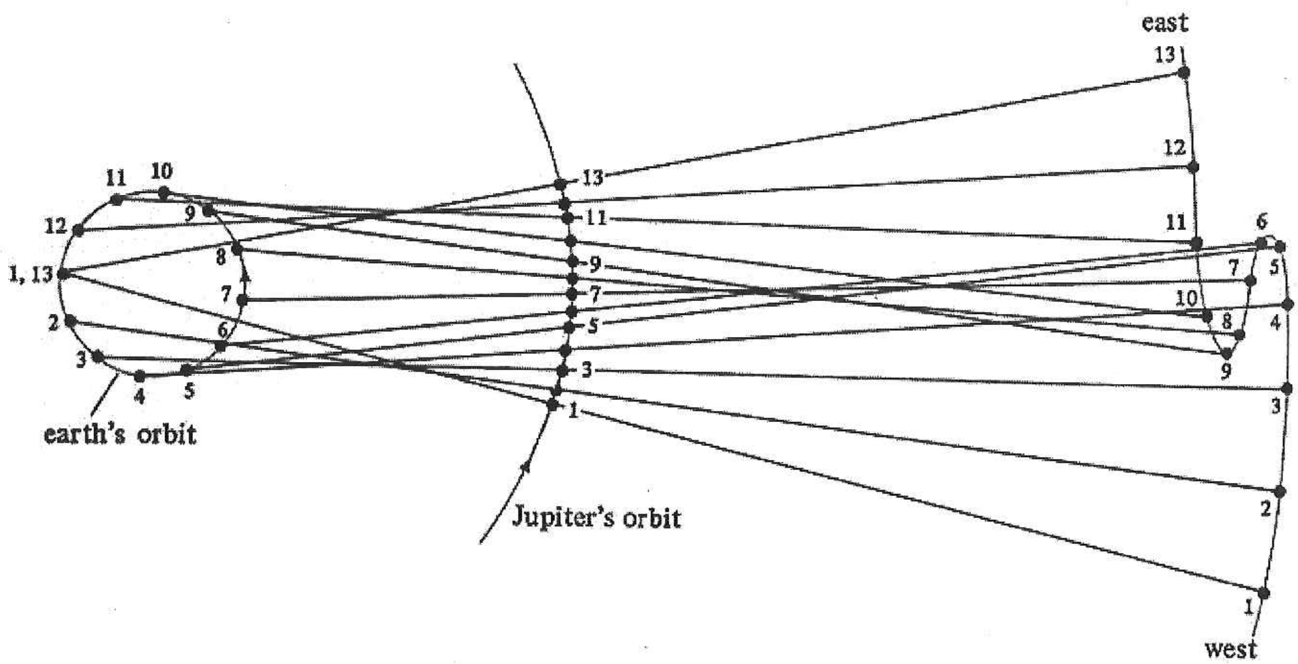
Get D_m by comparing the width of the earth's shadow to the moon during an eclipse of the moon.

(312-230 BC) – Aristarchus of Samos (continued)

- Important historic note; until Aristarchus, all models of our solar system were *geocentric*.
- Aristarchus was the first to introduce a *heliocentric* model of the known universe.
- Aristarchus also proposed that while the earth travels around the sun it spins on its axis.
- Nicolaus Copernicus would reinvent this theory eighteen hundred years later.

18 centuries

- The Aristarchus-Copernican model gives the simplest explanation of the motion of the planets.



As the earth and Jupiter rotate counterclockwise in the Copernican model, the line of sight from the earth to Jupiter rotates counterclockwise except for the motions from points 6 to 9. During the period 6–9, it appears to an observer on earth that Jupiter is moving westward relative to the stars.

- Unfortunately, the Aristarchus-Copernican model was not generally accepted.

(190 –120 BC) – Hipparchus

- Using Aristarchus' technique for finding the distance to the moon and sun, he redid Aristachus' measurements and found values for the distance to the moon very similar to the accepted values we use today, while his sun measurements were still inaccurate.
- Using centuries of celestial data he determined the axis of the earth is not fixed, but slowly precesses. He determined the time for each precession is 25,000 years.

MUCH LATER…

(150 AD) Ptolemy

- Argued against any model in which the earth moved because he thought the atmosphere would be left behind.
- Ptolemy developed a very complicated but accurate earth-centered model.
- Ptolemy's famous book, the "Almagest" provided a record of all the Greek models.
- The Almagest was used for more than 1000 years as the manual for determining the motion of the stars & planets.

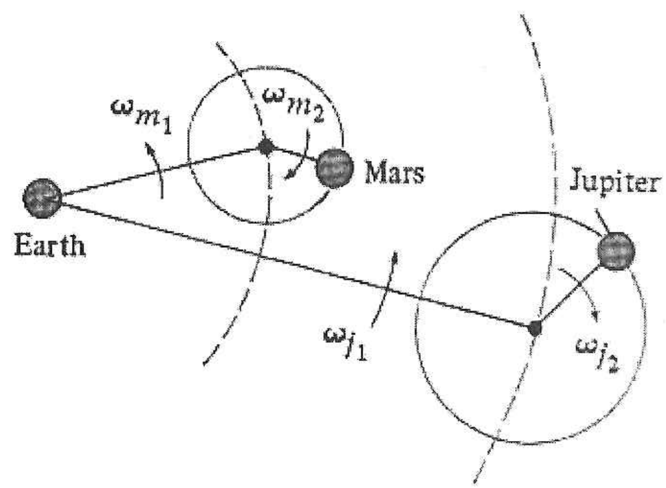


Figure 16-6 An earth-centered model for the motion of Mars and Jupiter with two radii of rotation for each planet. As seen from the earth, the apparent motion of the planets would not always be in the same direction.

HISTORY OF THE WORLD – PART II

The beginning of modern science

(1473 – 1543) Nicolaus Copernicus – Polish Monk

- Reinvented Aristarchus' Model of the heliocentric frame of reference for the solar system.
- He knew that the church had adopted Ptolemy's model as part of its dogma, and therefore would not accept this radical change.
- He *intentionally* delayed the publication of his great book "De Revolutionibus Orbium Coelestium" until the year he died, 1543.

(1564 –1643) Galileo Galilei – Italian

- Supported Copernicus model and used his newly invented "telescope" to help verify the model by observing the phases and apparent changes in size of Venus.

(1546 – 1601) Tycho Brahe – Danish nobleman

- Took years and years of very accurate data (pre-telescope) that he made with huge 20 ft protractors that he had built for sighting the directions to the heavenly bodies.
- He made records of his measurements on equally large spheres.

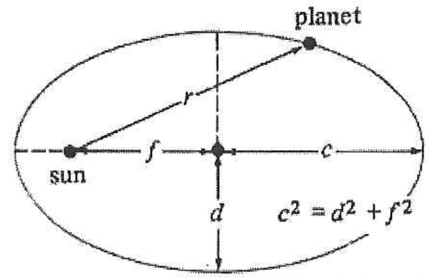
(1571 – 1630) Johannes Kepler – German

"The wandering mathematician"

- Kepler had the mathematical talent to use Tycho Brahe's data, and put it to good use. However, it is unclear if Kepler was given Tycho's data or whether he stole it.
- In either case he used it well, and the rest is history.

KEPLER'S THREE LAW'S OF PLANETARY MOTION

1st – The orbits of the planets are ellipses, with the sun at one focal point.



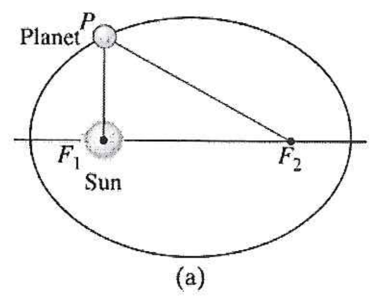
The orbit of the planet is an ellipse with the sun at a focal point. The distance c is called the semimajor axis of the ellipse.

2nd – The line joining the sun and a planet (its radius vector) sweeps over equal areas in equal times.

3rd – The squares of the periods of the planets' motions are proportional to the cubes of the semimajor axes of their elliptical paths; that is c^3/T^2 is the same for **all** the planets, where T is the time for the planet to complete one orbit about the sun and c is defined in the figure at the bottom of the last page.

$$\frac{c_A^3}{T_A^2} = \frac{c_B^3}{T_B^2}$$

- Galileo used his telescopes to discover the four moons of Jupiter and times their periods of orbit.
- He found that for all four moons Kepler's ratio of c^3/T^2 was constant.
- This convinced most scientists that Kelper's laws were not an accidental fit, and Copernicus' heliocentric universe model must be correct.



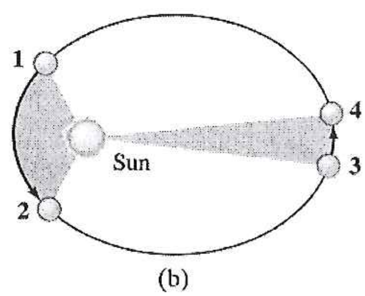


FIGURE 5–28 (a) Kepler's first law. An ellipse is a closed curve such that the sum of the distances from any point P on the curve to two fixed points (called the foci, F_1 and F_2) remains constant. That is, the sum of the distances $F_1P + F_2P$ is the same for all points on the curve. A circle is a special case of an ellipse in which the two foci coincide, at the center of the circle. (b) Kepler's second law. The two shaded regions have equal areas. The planet moves from point 1 to point 2 in the same time as it takes to move from point 3 to point 4. Planets move fastest in that part of their orbit where they are closest to the Sun.

- The Catholic Church was not convinced and Galileo was arrested for his teachings and excommunicated from the church. He has since been re-instated.

Kepler and Galileo had worked to show ***how*** the planets behaved, and how the motions of the moons of Jupiter and the planets of our sun obeyed the same rules. It was Newton's turn to explain ***what causes planets to obey the same rules***.

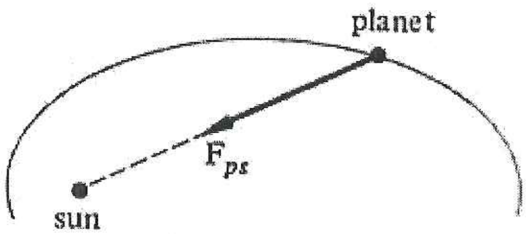
Newton's Analysis of Planetary Motion

(1642 – 1727) Isaac Newton

- 1666 – Using mathematics, Newton showed that for planets to move in elliptical paths.

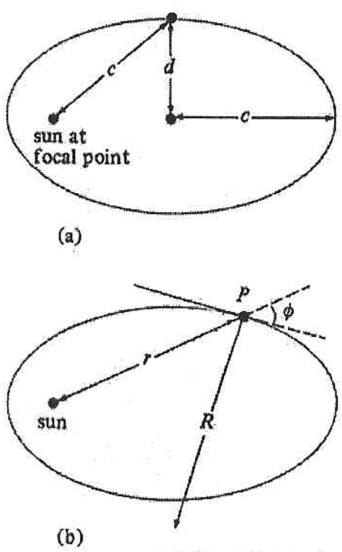
Newton proposed the following proportionality between the force, F and the distance between the centers of the planet and the center of the sun, d.

$$F \propto \frac{1}{d^2}$$



The force, $\mathbf{F}_{ps}$, on the planet is directed toward the sun.

Deriving Newton's Universal Law

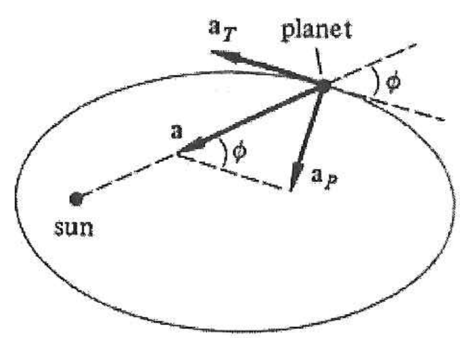


(a) The elliptical path of a planet with a semimajor axis c and a semiminor axis d. (b) The same path showing the radius of curvature at one point.

1. $\sum F = ma$ (Newton's 2nd Law)
2. $F_{P,S} = m_P a_P$ ($F_{P,S}$ =Force on the Planet by the Sun)
3. $a_P = \left(\dfrac{4\pi^2 c^3}{T^2}\right)\dfrac{1}{d^2}\sin\phi$

Looking at the picture below you can see that the total acceleration is given by $a = a_P/\sin\phi$ and therefore the above equation simplifies.

4. $a = \left(\dfrac{4\pi^2 c^3}{T^2}\right)\dfrac{1}{d^2}$ (If we Subsitute Eq. 4 into Eq. 2)
5. $F_{P,S} = \dfrac{M_P}{d^2}\left(\dfrac{4\pi^2 c^3}{T^2}\right)$



The acceleration of a planet.

Using Kepler's 3rd law of planetary motion Newton knew that

$\frac{c^3}{T^2}$ was a constant for all planets orbiting the sun. Therefore, to simplify Equ. # 5 Newton defined a new variable...

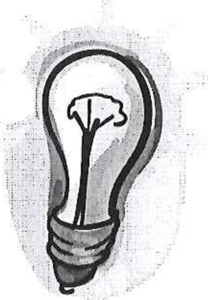
6. $\mathrm{K} \equiv \frac{4\pi^2 c^3}{T^2}$ which he calculated as $1.33 \times 10^{20} \frac{\mathrm{m}^3}{s^2}$

Subsituting Equ # 6 into Equ. # 5 we get

7. $F_{P,S} = K\frac{M_P}{d^2}$

Using his own 3rd Law of motion Newton realized that

$$F_{S,P} = F_{P,S} = K\frac{M_P}{d^2}$$



HERE IT COMES!!!
ANOTHER STROKE OF GENIUS FROM NEWTON

Newton sees symmetry between these forces.

- If $F_{S,P}$ is proportional to the mass of planet, then the $F_{P,S}$ must be proportional to the mass of the sun, and therefore sets

$K = GM_s$ where he called G the *Universal Gravitational Constant.*

With one last substitution Newton finds...

$$F_{S,P} = F_{P,S} = G\frac{M_s M_P}{d^2}$$ Realize Newton still did not know **G or M_S**

But, he did know $GM_S = 1.33 \times 10^{20} \frac{\mathrm{m}^3}{s^2}$

Newton assumed that his law should work for any two objects not just planets. ***Newton's Law of Universal Gravitation*** for any two objects is stated as follows…

$$\boxed{F = G\frac{m_1 m_2}{d^2}}$$

The direction of the force is along the line joining the two objects and is always an attractive force.

THE CAVENDISH EXPERIMENT (1798)
OR
The torsion balance and the measurement of the gravitational constant, "G".

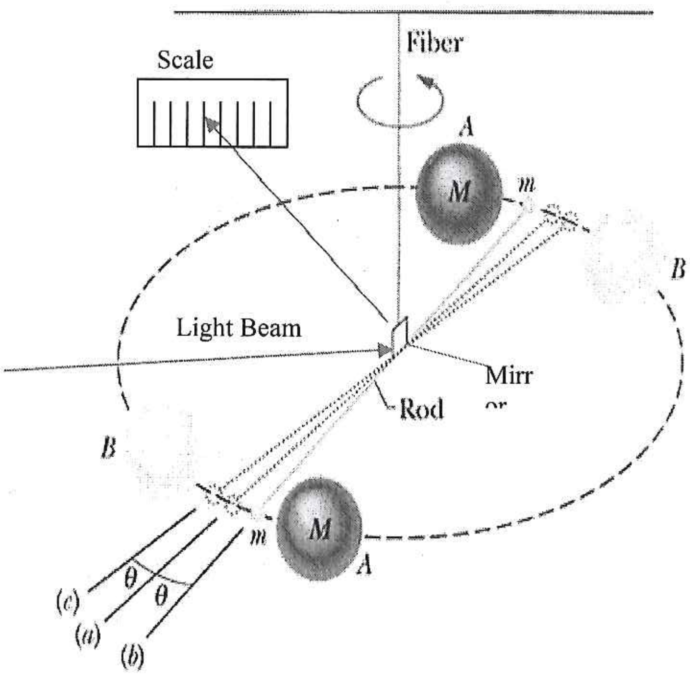


FIGURE 15-9 The apparatus used in 1798 by Henry Cavendish to measure the gravitational constant G. The large spheres of mass M, shown in configuration AA, can also be moved to configuration BB. (*a*) Orientation of the rod when the large spheres are absent and the fiber is not twisted. (*b*) Equilibrium orientation when large spheres are at AA. (*c*) Equilibrium orientation when large spheres are at BB. The angle θ is exaggerated.

Torsion Balance Principle of Operation

1. Two small masses are placed at the end of a light rod.
2. The force required to rotate the fiber from its untwisted position (a), is measured.
3. The force required to twist the fiber is calibrated as a function of the angle θ,

   $$F_{twist} \propto \theta_{measured}$$

4. The light beam and scale are used to magnify θ.
5. When the mass *M*, is brought near to mass *m* it applies a gravitational force onto the fiber. To check for symmetry it is placed in positions AA and BB.
6. Knowing that the force causing the twist was caused by gravity Cavendish now used The Law of Universal Gravitation. $F_{Twist} = F_{Gravity} = G\frac{mM}{d^2}$
7. Cavendish was now able to isolate G, and calculated it as…

$$G = 6.67x10^{-11}\,\frac{N\cdot m^2}{kg^2}$$

EXAMPLE PROBLEM: Assume the earth is moving in a circular orbit around the sun. Using the following data calculate the speed of the earth in its orbit in miles/hr.

Mean Radius of Orbit = 1.5×10^{11} m   1 mile = 1.61 km

Mass of Sun = 1.99×10^{30} kg $G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$

$r = 1.5 \times 10^{11}$ m

$m_s = 1.99 \times 10^{30}$ kg

$$F_g = G \frac{m_E m_s}{d^2}$$

Since F_g is causing this planet to travel in a circle, $F_G = F_c$

$$G \frac{\cancel{m_E} m_s}{d^2} = \frac{\cancel{m} v^2}{r}$$

This is the mass of the object in circular motion. Here, its the earth

$$G \frac{m_s}{d^2} = \frac{v^2}{r}$$ ← $r = d$ in this case

$$\therefore v = \sqrt{G \frac{m_s}{r}} = \sqrt{\frac{6.67 \times 10^{-11}(1.99 \times 10^{30})}{1.5 \times 10^{11}}}$$

$$\boxed{v = 29{,}747 \text{ m/s} = 66{,}515 \text{ mi/hr}}$$

Check:
How close could we have gotten before this?

$$v = \frac{2\pi r}{T} = \frac{2\pi(1.5 \times 10^{11})}{31{,}557{,}600 \text{ sec}} = \boxed{29{,}865.32 \text{ m/s}}$$

12. (I) Calculate the force of gravity on a spacecraft 12,800 km (2 earth radii) above the Earth's surface if its mass is 1400 kg.

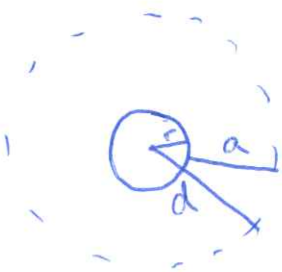
$m_s = 1400 kg$

$m_e = 5.97 \times 10^{24} kg$

$d = r + a = 3 r_{earth}$

$$F_g = G \frac{m_s m_e}{d^2}$$

$$= 6.67 \times 10^{-11} \left[\frac{1400(5.97 \times 10^{24})}{19{,}200{,}000^2} \right]$$



$d = 12{,}800{,}000 m + 6{,}400{,}000 m$

$d = 19{,}200{,}000 m$

$$\boxed{F_g = 1512.26 N}$$

* NOTE: $F = ma$

$1512.26 = 1400a \rightarrow a = 1.080 m/s^2$

This is 1/9 that of earth! Since distance is 3 times as far, a is 1/9 as strong!

13. (I) A hypothetical planet has a mass 2.5 times that of Earth, but the same radius. What is *g* near its surface?

$m_p = 2.5 m_E$

$r = 6.38 \times 10^6 m$

$$F_g = G \frac{m_p \cancel{m_o}}{d^2} = \cancel{m_o} a$$

The mass of the object doesn't affect the acceleration!

$$\frac{6.67 \times 10^{-11}(2.5(5.97 \times 10^{24}))}{(6.38 \times 10^6)^2} = a$$

$$\boxed{a = 24.47 m/s}$$ ← This is 9.8*2.5!

Satellite Motion:
Imagine placing a large cannon on top of Mount Everest.
If we were to load the cannon with enough gun powder and fire it HORIZONTALLY to the surface of the Earth below, we may hit a town far away.
If we were to load the cannon with more gun powder and fire it HORIZONTALLY to the surface of the Earth below, we may even hit a different country far away.
Let's say we were to load the cannon with just enough gun powder and fire it HORIZONTALLY to the surface of the Earth below, we may even hit a country halfway around the world.
Well, let's say we were to load the cannon with perfect amount of gun powder and fire it HORIZONTALLY to the surface of the Earth below, so that it just keeps falling for ever as the Earth curves away from its fall. If this cannon ball does not hit the back of the cannon which it was fired from, then this cannon ball will continuously go around the world; falling towards the Earth… FOREVER!!!!!

This is exactly what is happening to our moon with Earth and all the planets with our Sun.

We can calculate the Speed required!

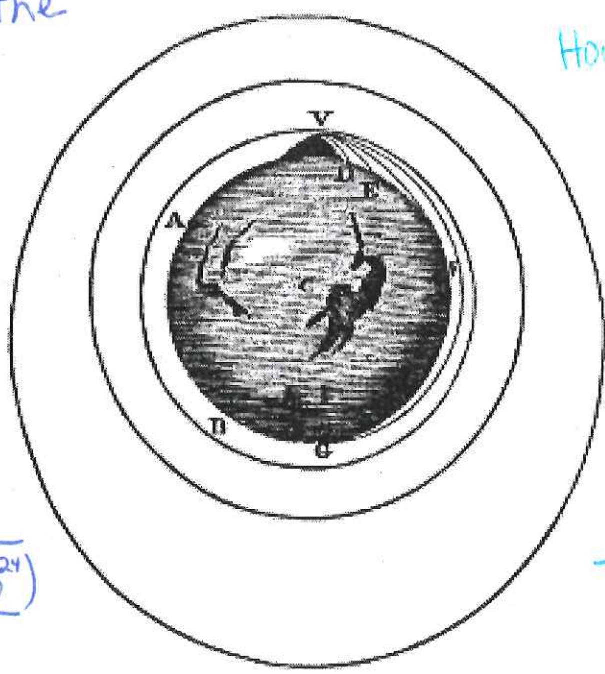
$F_c = F_G$

$\frac{\cancel{m} v^2}{\cancel{r}} = G\frac{m_e \cancel{m}}{r^{\cancel{2}}}$

$\boxed{v = \sqrt{G\frac{m_e}{r}}}$

$= \sqrt{\frac{6.67\times10^{-11}(5.98\times10^{24})}{6.38\times10^{6}}}$

$\boxed{v = 7906.8\ m/s}$



How Long will it take for one orbit?

$v = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v} = \frac{2\pi(6.38\times10^{6})}{7906.8}$

$T = 5069.88$ seconds

$= 84.498$ minutes

$\boxed{T = 1.41\ \text{hours}}$

Orbiting Velocity and Escape Velocity:

RECALL:

$$\sum F_c = ma_c = \frac{mv^2}{r}$$

And

$$\boxed{F_G = G\frac{m_1 m_2}{r^2}}$$

$$F_C = F_G$$

$$\frac{m_{sat} v^2}{r} = G\frac{M_{Earth} m_{sat}}{r^2}$$

$$v_{orbiting} = \sqrt{G\frac{M_{Earth}}{r}}$$

$$v_{escape} > \sqrt{G\frac{M_{Earth}}{r}}$$

These are the equations to use for orbiting and escape velocities.

A 5000.0 kg satellite is moving in a stable circular orbit at altitude of 12,800 km above the earth's surface.

$\boldsymbol{R}_{earth}$= 6.38 x 10^6 m $\boldsymbol{M}_{earth}$= 5.98 x 10^{24} kg

$\boldsymbol{G}$= 6.67 x 10^{-11} N*m²/kg²

a. Please calculate the orbiting velocity of the satellite.

$$V = \sqrt{\frac{Gm_e}{r}} = \sqrt{\frac{6.67\times10^{-11}(5.98\times10^{24})}{(6.38\times10^6+12.8\times10^6)}} = 4560.25\,m/s$$

a: 4560.25 m/s

b. What is its period (time to make one orbit) in hours.

$$V = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi r}{V}$$

$$T = \frac{2\pi(6.38\times10^6+12.8\times10^6)}{4560.25}$$

$$T = 26426.51\,sec$$

b: 7.3 hours

A Geosynchronous satellite is a satellite that has a period of 1 Earth day so that it is stationary to one fixed location on the surface of the earth. What must be the **altitude in terms of radius of the Earth** and the **orbiting velocity** for this satellite **in miles per hour**?
Please show your work…

Altitude = 6.6 ***R**earth*

velocity = 6870.28 **miles/hour**

$T = 24 \text{ hrs} = 86400 \text{ seconds}$

$v = \frac{2\pi r}{T}$ $\qquad$ $v = \sqrt{G\frac{m_E}{r}}$

$$\frac{2\pi r}{T} = \sqrt{G\frac{m_E}{r}}$$

$$\frac{4\pi^2 r^2}{T^2} = G\frac{m_E}{r}$$

$$r^3 = \frac{G m_E T^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{6.67\times10^{-11}(5.98\times10^{24})(86400)^2}{4\pi^2}}$$

$$r = \boxed{42250474.31\,m} \times \frac{1\,R_{earth}}{6.38\times10^6\,m} = \boxed{6.6\,R_{earth}}$$

$$v = \frac{2\pi(42250474.31)}{86400} = \boxed{3072.5\,m/s} \times \frac{1\,mi}{1610\,m} \times \frac{3600\,s}{1\,hr}$$

$$\boxed{v = 6870.28\,mph}$$

14. (II) You are explaining to friends why astronauts feel weightless orbiting in the space shuttle, and they respond that they thought gravity was just a lot weaker up there. Convince them and yourself that it isn't so by calculating how much weaker gravity is 300 km above the Earth's surface.

$d = r + a = 6.38 \times 10^6 m + 300{,}000 m = 6{,}680{,}000 m$

$m_e = 5.97 \times 10^{24} kg$

$F_g = \cancel{m}g = G \frac{\cancel{m_1} m_2}{d^2}$

$g = \frac{6.67 \times 10^{-11} (5.97 \times 10^{24})}{6{,}680{,}000^2}$

$\therefore \boxed{g = 8.9 m/s^2}$ ← not much weaker!

15. A projected space station consists of a circular tube that is set rotating about its center (like a tubular bicycle tire) (Fig. 5-42). The circle formed by the tube has a diameter of about 1.1 km. (a) On which wall inside the tube will people be able to walk? (b) What must be the rotation speed (revolutions per day) if an effect equal to gravity at the surface of the Earth (1.0 g) is to be felt?

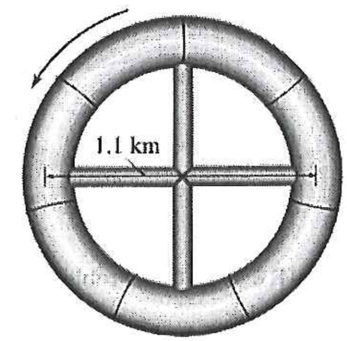


FIGURE 5-42

$r = .55 km = 550 m$

a. Outside wall

b. $F_N = \frac{\cancel{m} v^2}{r} = \cancel{m} g$

$$v = \sqrt{gr} = 73.42 \, m/s$$

$$\frac{73.42 m}{sec} \times \frac{3600 s}{1 hr} \times \frac{24 hr}{1 day} \times \frac{1 rev}{2\pi r \, m} = \boxed{1835 \, rev/day}$$

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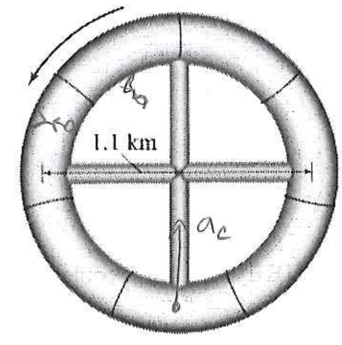

FIGURE 5-42

$r = .55\,\text{km} = 550\,\text{m}$

a. OUTSIDE WALL → NORMAL FORCE PROVIDES THE CENTRIPITAL ACCELERATION.

② SOLVE THAT!

$$\left[\frac{\cancel{m}v^2}{r} = G\frac{\cancel{m}M}{r^2} = \cancel{m}a\right]$$

b. ① $F_N = \frac{\cancel{m}v^2}{r} = \cancel{m}g$

$v^2 = \sqrt{gr} = 73.42\,\text{m/s}$

$\frac{73.42\,\text{m}}{\text{s}} \times \frac{3600\,\text{s}}{1\,\text{hr}} \times \frac{24\,\text{hr}}{1\,\text{day}} \times \frac{1\,\text{rev}}{2\pi r\,\text{m}} = \boxed{1835\,\text{rev/day}}$

$$\frac{\left(\frac{2\pi r}{T}\right)^2}{r} = Gm = a$$

$$\frac{4\pi^2 r}{T^2} = a$$

$$\frac{4\pi^2(550)}{9.8} = T^2$$

$$T = 47.07\,\text{sec/rev}$$

$\frac{.0212\,\text{rev}}{\text{sec}} \times \frac{3600\,\text{sec}}{1\,\text{hr}} \times \frac{24\,\text{hr}}{1\,\text{day}} = \boxed{1835\,\text{rev/day}}$